Fakir Mohan Autonomous College, Balasore

UG II Year , CC - paper - V
Theory of real functions

July 20, 2021

Questions carrying one mark each :-

1. Find the limit of the function at $x = 0$,

$$f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$$

2. Find

$$\lim_{x \to 0} \frac{1}{x} \sin(\frac{1}{x}) \ .$$

3. Define Cauchy's mean value theorem.
4. Define Taylor's theorem as Lagrange's form of remainder .
5. Find the sum of the series

$$\frac{1}{3} + \frac{2}{4} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{724} + ... =?$$

6. Find the Macaurin's series for $\ln(\frac{1+x}{1-x})$.
7. When a function $f(x)$ is said to be Riemann integrable ?
8. Show that the constant function k is integrable.
9. Is

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \\ 1, & x \in \mathbb{Q}^c \end{cases} \quad R - integrable \ ?$$

10. What is $\Gamma(\frac{1}{2})$?
11. Define pointwise convergence of a sequence of function $\{f_n\}$.
12. Define uniform convergence of a sequence of function $\{f_n\}$.
13. Is $f_n(x) = nx(1 - x^2)^x, 0 \leq x \leq 1, n = 1, 2, 3, ...$ uniform convergent ?
14. Define Cauchy's criterion for uniform convergence.
15. Is $f_n(x) = \frac{1}{x+n}$ uniformly convergent ?

Questions carrying two mark each :-

16. Show that the integral

$$\int_0^{\frac{\pi}{2}} (\frac{\sin^m(x)}{x^n})dx$$

exists if and only if $n < m + 1$.

17. Show that $\int_0^1 (\frac{\log x}{\sqrt{x}})dx$ is convergent.

18. Give an example of function which is not Reimann integrable . Justufy your answer.

19. If f is continuous and positive on $[a, b]$ then show that $\int_a^b f dx$ is also positive.

20. Find
$$\lim_{x \to 0} (\frac{(1+x)^{\frac{1}{x}} - e}{x}).$$

21. Find the maxima and minima of the function $f(x) = x^{\frac{3}{2}}$.

22. If $f(x) = 2x^3 - 9x^2 - 7$, find the interval where the function is one-one.

23. State Rolle's theroem and its geometric interpretation.

24. Determine the radius of convergence of the power series
$$\sum \frac{(n!)^2 x^{2n}}{(2n)!}.$$

25. Compute
$$\int_0^\infty e^{-\frac{x^2}{2}} dx?$$

Subjective Questions :

26. Find the radius of convergence and the exact interval of convergence of
$$\sum \frac{1.2.3....n}{1.3.5.....(2n-1)} x^{2n}.$$

27. The sum of the infinite series
$$S = \frac{1}{2} - \frac{1}{3 \times 1!} + \frac{1}{4 \times 2!} - \frac{1}{5 \times 3!} + ...?$$

28. State and prove Cauchy's mean value theorem .

29. Prove that a bounded function f is integrable on $[a, b]$ iff for every $\epsilon > 0$ there exists a partition P of $[a, b]$ such that $U(f, P) - L(f, P) < \epsilon$.

30. Show that the improper integral
$$\int_a^b \frac{dx}{(x-a)^n}$$
converges iff $n < 1$.

31. Show that the sequence f_n , where $f_n(x) = x^n$ is uniformly converges on $[0, k], k < 1$ and only pointwise converges on $[0, 1]$.

32. State and prove Abel's test for uniform convergent.

33. Show that $\int_0^1 \frac{\sin(\frac{1}{x})}{x^p} dx$, convergent absolutely for $0 < p < 1$.

34. Prove that every continuous function is integrable.

35. Give an example of a function where set of points of discontinuties are integers, check the Reimann integrability of the function.

.... Good Luck ....